

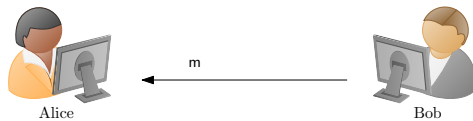
# Short Identity-Based Signatures with Tight Security from Lattices

Jiaxin Pan  NTNU

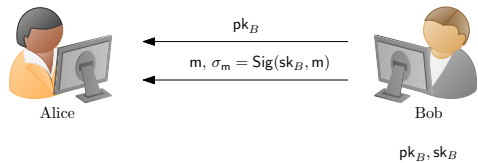
Benedikt Wagner  KIT  
Karlsruhe Institute of Technology

# Digital Signatures in Practice

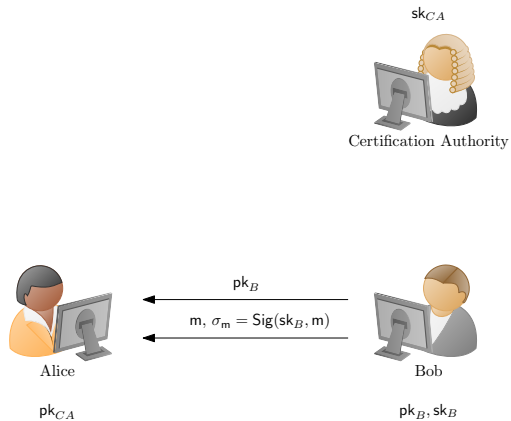
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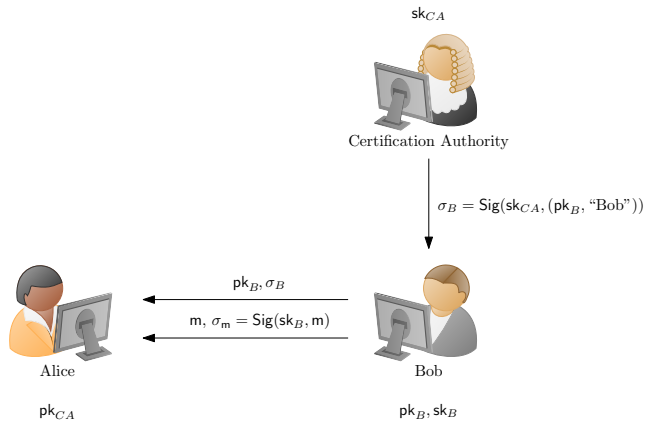
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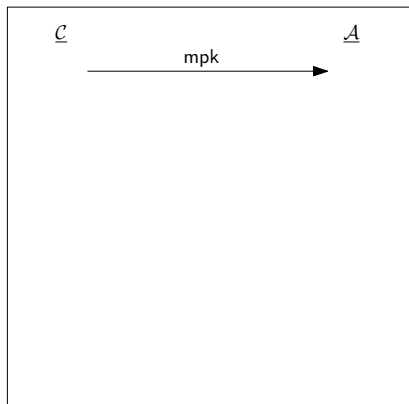
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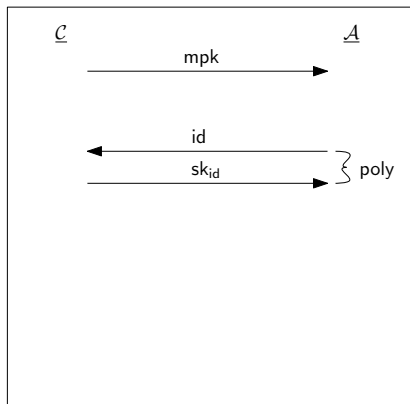
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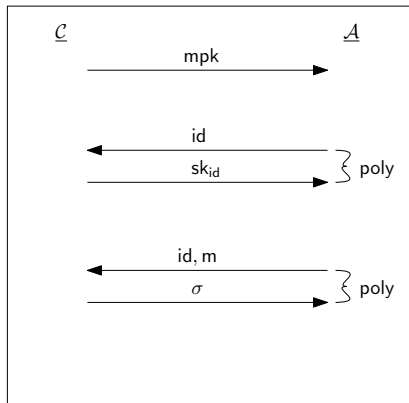
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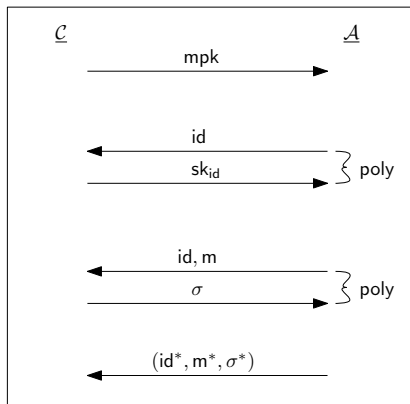
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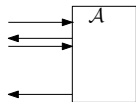


$\mathcal{A}$  wins iff  $\text{id}^* \text{ fresh} \wedge (\text{id}^*, m^*) \text{ fresh} \wedge \text{Ver}(\text{mpk}, \text{id}^*, m^*, \sigma^*)$

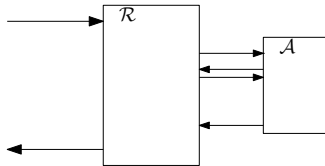
# The Typical Proof Strategy



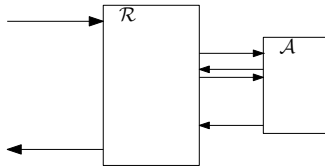
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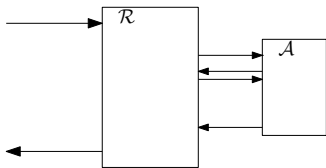


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$A$  breaks IBS with probability  $\epsilon_A$   
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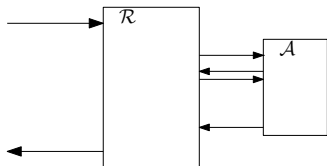
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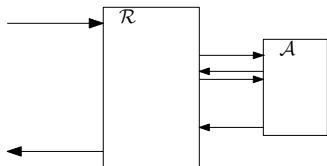
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## Tightness

We say the reduction is tight, iff the loss  $L$  is a small constant.

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⇒ **avoid certification approach**

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# Overview

SIS

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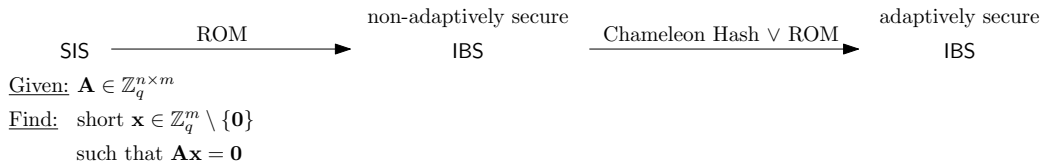
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- allows to derive trapdoor for any extension  $[\mathbf{A} \mid \mathbf{B}]$

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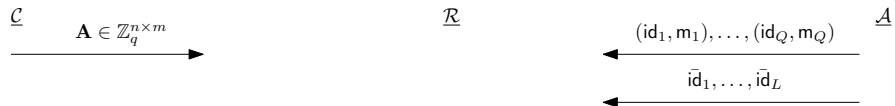


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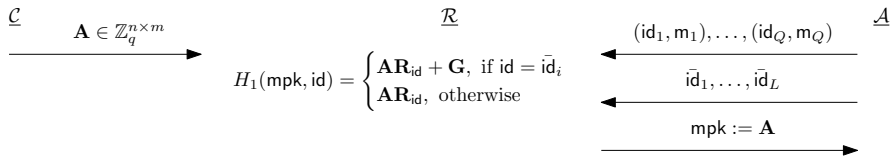
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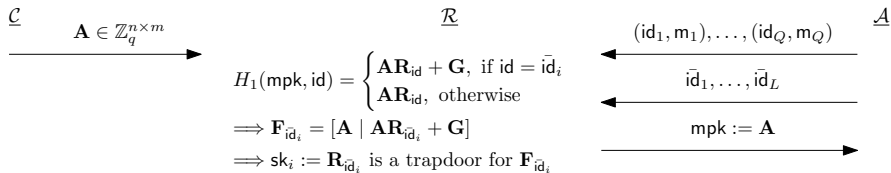
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$$\begin{array}{c} \xleftarrow{(\text{id}_1, m_1), \dots, (\text{id}_Q, m_Q)} \underline{\mathcal{A}} \\ \xleftarrow{\bar{\text{id}}_1, \dots, \bar{\text{id}}_L} \\ \xrightarrow{\text{mpk} := \mathbf{A}} \end{array}$$

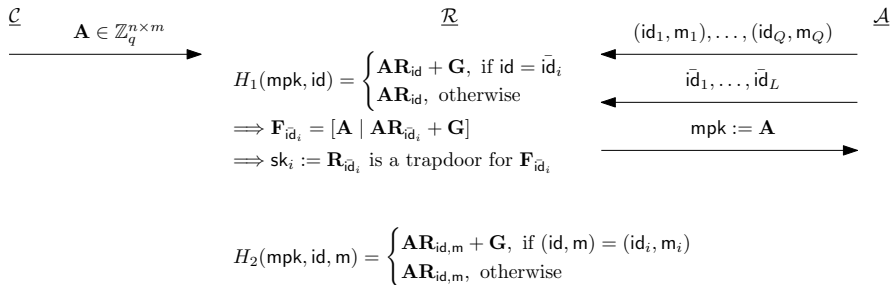
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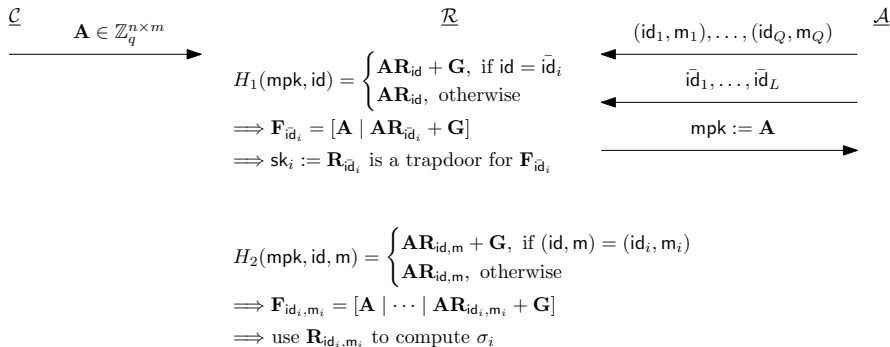
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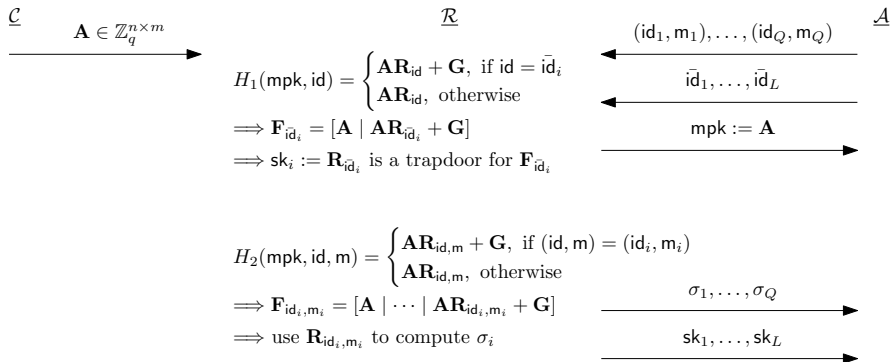


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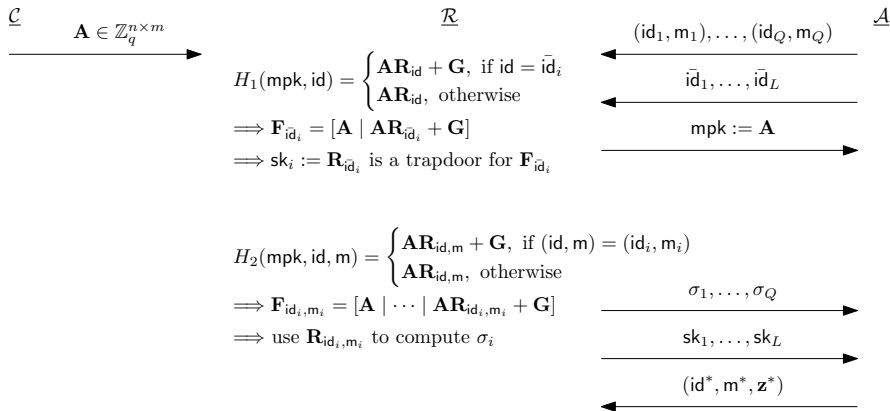




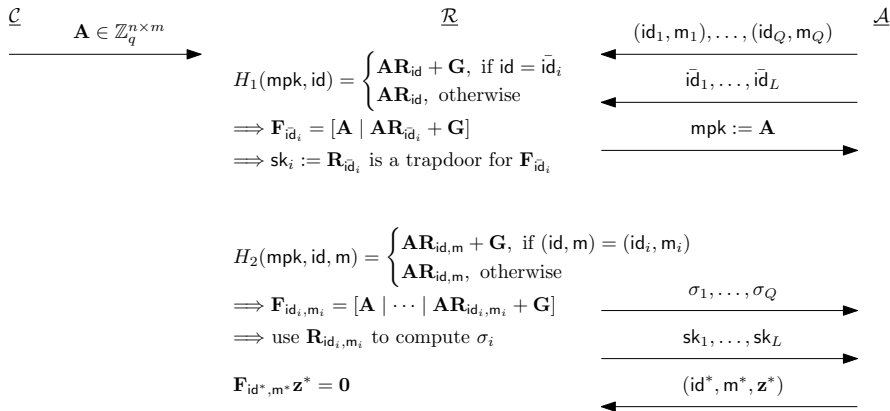
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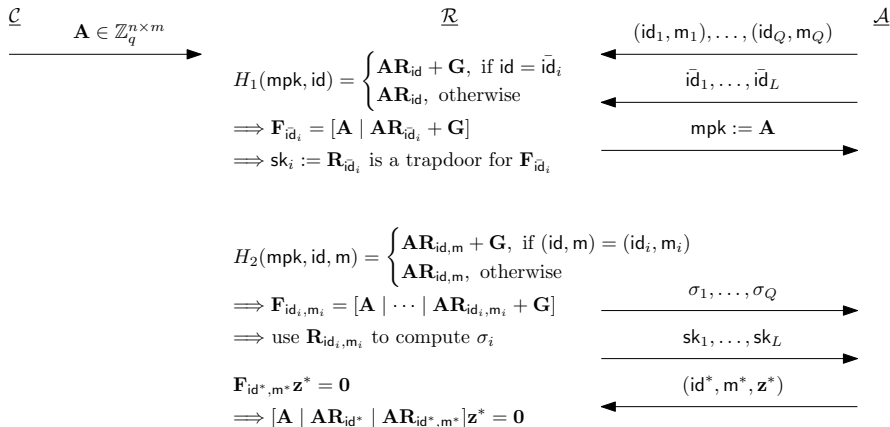
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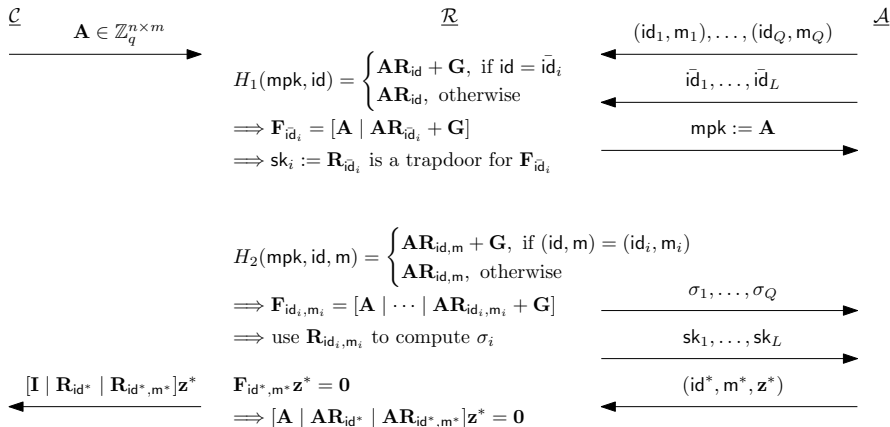
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Thank you for your attention!